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Cooperative allocation and guidance for air defence application

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ABSTRACT

This project proposes a centralised algorithm to design cooperative allocation strategies and guidance laws for air defence applications. Scenarios in naval and ground context have been defined for performance analysis by comparison to a benchmark target allocation policy. The cooperative target allocation algorithm is based on the following features: No Escape Zones (differential game NEZ) computation to characterise the defending missile capturability characteristics; In Flight (re) Allocation (IFA algorithm, late committal guidance) capability to deal with target priority management and pop up threats; capability to generate and counter alternative target assumptions based on concurrent beliefs of future target behaviours, i.e. Salvo Enhanced No Escape Zone (SENEZ) algorithm. The target trajectory generation has been performed using goal oriented trajectory extrapolation techniques. The target allocation procedure is based on minimax strategy computation in matrix games.

1. Introduction

This research programme has focused on the problem of naval-based air defence systems which must defend against attacks from multiple targets. Technology developments in the field of modular data links may allow the creation of a multi-link communication network to be established between anti-air missiles and the launch platform. The future prospect of such ad hoc networks makes it possible to consider cooperative strategies for missile guidance. Many existing guidance schemes are developed on the basis of one-on-one engagements which are then optimised for many-on-many scenarios (Ge, Tang, Reimann, & Vachtsevanos, 2006; Jang & Tomlin, 2005). A priori allocation rules and natural missile dispersion can allow a salvo of missiles to engage a swarm of targets; however, this does not always avoid some targets leaking through the salvo, whilst other targets may experience overkill.

Cooperative guidance combines a number of guidance technology strands such as:

- Prediction of the target behaviour.
- A mid-course guidance to place the missile in position to acquire and engage the target.
- Allocation/re-allocation processes based on estimated target behaviour and no escape zones.
- Terminal homing guidance to achieve an intercept.

These strands have been and these have been studied as part of the research programme. In the terminal phase, guidance has been achieved by handover to the linear differential game (LDG) guidance law (Shinar & Shima, 2002). Two approaches to missile allocation have been considered (Shin, et al., 2010a). This paper focus on the second one exploiting the no escape zones (NEZ, Isaacs, 1967) defined by a LDG guidance law which either acts to define an allocation before launch (ABL) plan or refine an earlier plan to produce an in-flight allocation (IFA) plan.

One of main challenges in the air defence is unpredictability on future manoeuvres of the oncoming threats. Often, oncoming threats deceive air defence systems by pretending to head to a different direction at the initial stage and changing their course at the late stage. Most of existing guidance algorithms repulsively react to the oncoming threats based on their current motion information. This approach cannot effectively cope with the unpredictability of the oncoming threat manoeuvres. Therefore, this paper aims to address this problem and develop an effective solution.

Despite it is difficult to accurately predict future manoeuvres of the oncoming threat, what is certain is that the threat will either

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fly straight, or turn. Based on this certainty, the potential future trajectories of the threat can be partitioned into a several geometric hypotheses. Under assumption that several missiles can be launched together, this paper develops cooperative allocation and guidance scheme which can intercept the oncoming threats against all these trajectory hypotheses. This approach will allow the proposed scheme to effectively intercept oncoming threats whose manoeuvres are unpredictable.

In Section 2, a statement of the problem is given and the proposed SENEZ concept is described. Then, the proposed target allocation algorithm is detailed in Section 6 after introducing essential concepts used in the target allocation from Sections 3 to 5. Missile guidance, both mid-course and terminal, is discussed in Section 7. The simulation conditions and benchmark allocation policy are addressed at the beginning of Section 8. The simulation results of the SENEZ algorithm from a Simulink 6DOF model are also discussed in Section 9. Finally, Sections 9 and 10 conclude this study and remark on consideration for the exploitation of these cooperative guidance technologies.

2. SENEZ concept

There are occasions when the weapon system policy for defending against oncoming threats involves firing two or more missiles at the same target. Fig. 1 shows an example of such engagements where three missiles are launched against two possible oncoming threats.

Without any action taken, the missiles will naturally disperse en-route to the target, each arriving at the point of homing with a slightly different geometry. In such a case, there will be a significant overlap of the NEZ. In this study, a salvo enhanced no escape zone (SENEZ) concept was introduced to efficiently manage this type of engagement, with the cooperating missiles increasing their chances of at least one missile intercepting the target. Mathematical analysis of engagements involving only two defending missiles and one threat have been also performed (Ganebny, Kumkov, Le Ménec, & Patsko, 2012). SENEZ is an attempt to deal with still more realistic scenarios and more players.

In the naval or ground application, it is often the case that a number of assets may be situated in close vicinity to each other. In this situation, it may be difficult to predict which asset an inbound threat is targeting. In the case of air-to-air engagements, there are various break manoeuvres, which an oncoming threat such as an aircraft or attacking missile could execute to avoid an interceptor. Although it is difficult to predict which guidance algorithm the oncoming threat uses or for which defended asset it heads, it is apparent that the threat will either fly straight or turn. Based on this obvious fact, it is possible to partition possible future trajectory of the threat into a small number of bundles. For example, if the oncoming threat is predicted mostly to fly straight, its predicted future trajectory can partitioned into a front sector; if mostly to turn left, into a left sector; and if mostly to turn right, then into a right sector. In this study, the number of the trajectory bundles is determined by the number of missiles in the salvo. Fig. 2 illustrates the partition of possible future paths of the threats for an engagement scenario where the number of missiles in salvos is up to three.

Most of existing guidance algorithms repulsively generate guidance commands based on the current motion information of the oncoming target. However, when there exist a number of defending missiles launched together, a possible effective strategy

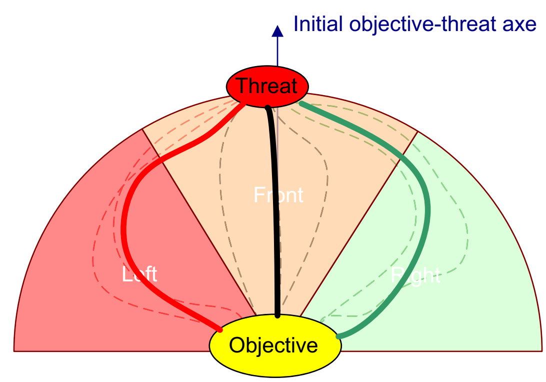

Fig. 2. Trajectory partition into three sectors: the number of missiles in the salvo is three.

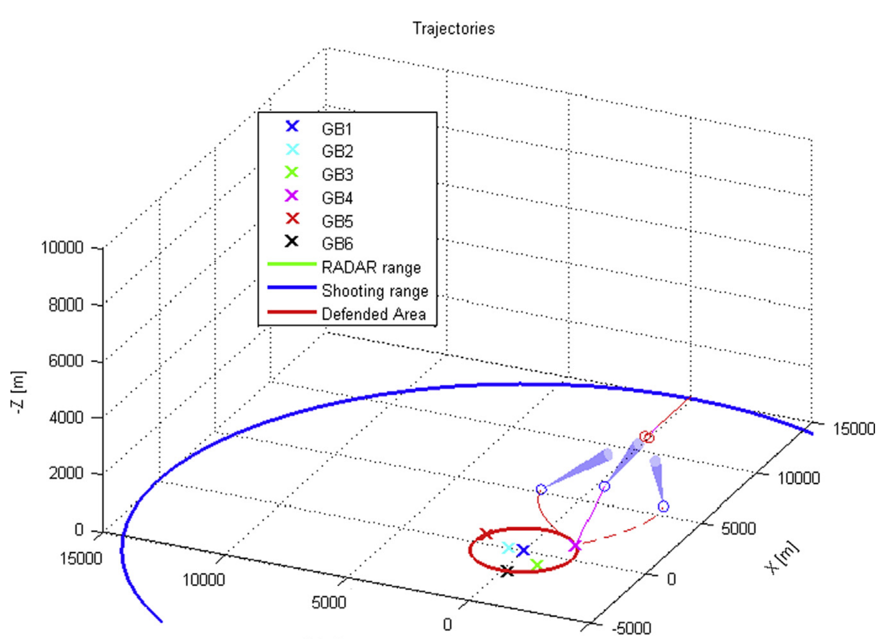

Fig. 1. Multi shoot example in SENEZ firing policy.

is to develop guidance against the partitioned possible future paths of the oncoming threats. By selecting well chosen geometric paths it should be possible to direct the defending missiles in such a way that each partition of the possible target trajectory bundles falls within the no escape zone (NEZ) of at least one missile. Consider a naval case of a two missile salvo, and a threat that is initially heading straight towards the launch vessel; there is a possibility that the threat may break left or right at some point. One defending missile can be directed to cover the break right and straight-on possibilities; the second missile would defend against the break left and straight-on possibilities. By guiding to bundle partitions prior to the start of homing, the NEZ of the firing is enhanced. At least one of the missiles will be able to intercept the target. This SENEZ firing policy differs from the more standard shoot-look-shoot policy which considers the sequential firing of missiles where a kill assessment is performed before firing each new missile launch.

3. Goal oriented target prediction

Different approaches have been studied to predict target positions (Shin et al., 2011). Results detailed in the following are based on the version implementing the goal oriented approach; which is based on the hypothesis that the target will guide to a goal.

As mentioned in the previous section, the target trajectories have been classified into three categories: threats coming from the left, from the front and from the right with respect to the objective (refer to Fig. 2). If the oncoming threat is assumed to use the conventional guidance law, it is impossible to generate trajectories fall into these three sectors. Therefore, we first generate waypoints corresponding to each target trajectory class. Then, we compute the target trajectories that lead to the threat passing by the waypoints based on trajectory shaping guidance (TSG) (Zarchan, 2007).

The basic TSG is similar to PN (Proportional Navigation) with a constraint on the final line-of-sight (LOS) angle in addition. This means that near impact, the LOS angle λ equals a desired value λ_F. A 3D version of this law is applied from the threats initial position to the waypoint. When the waypoint is reached, a switch is made from TSG to standard PN to guide on the objective. The LOS final angle of the TSG law is chosen to bring the threat aiming directly at the objective when it reaches the waypoint. Fig. 3 illustrates how assumed target trajectories have been generated.

A set of three waypoints per target is defined using polar parameters (angle Ψ_{wpt} and radius R_{wet}). All waypoints belong to a circle of radius R_{wet} centred on the supposed objective. Waypoints are then spread with Ψ_{wpt} as an angular gap, using the initial objective-threat line as a symmetry line defined at RADAR detection. In this way there is one trajectory per hypothesis, as seen in the Fig. 4.

Waypoints are defined for each target depending on its position at the time it is detected. To avoid a high disturbance of the defending missiles guidance, it is assumed that these waypoints do not change as the engagement evolves as shown in Fig. 5.

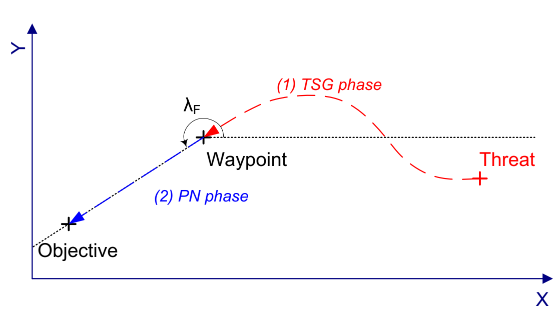

Fig. 3. 2D target trajectory generation using waypoints, TSG and PN as terminal homing guidance.

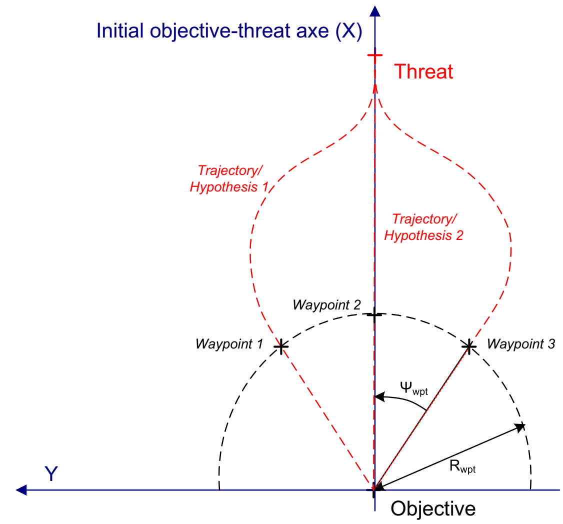

Fig. 4. Waypoints geometry for target trajectory generation.

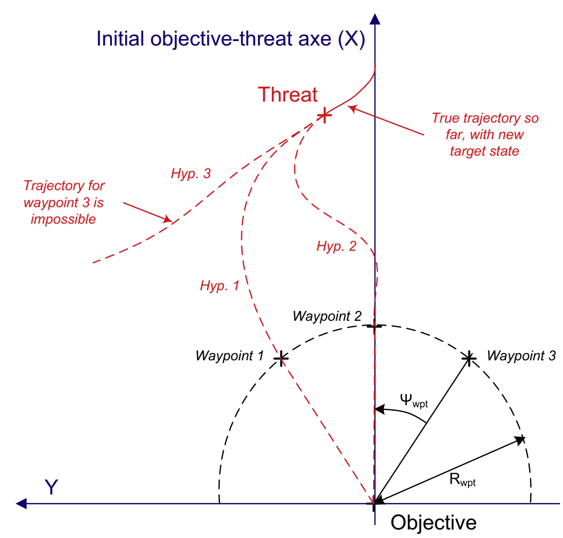

Fig. 5. Evolution of the engagement; waypoints do not move; waypoints trajectories become impossible.

Some hypotheses will become progressively less likely to be true and others appear to be a good approximation of reality. In due course, some hypotheses will become unachievable and will be discarded during the cost computation process.

4. Predicted NEZ

The SENEZ target allocation algorithm is in charge of evaluating all missile-threat-hypothesis engagements (Le Ménec, 2009; Le Ménec et al., 2009). This means that the algorithm must be able to tell for each case if successful interceptions are possible and to give a cost on a scale that enables comparisons. Usage of the following letters is now reserved:

- W is the number of waypoints considered.
- N is the number of defensive missile that can be allocated to a target (i.e. that are not already locked on a target, or destroyed).
- P is the number of active and detected threats.

Table 1
Costs over target trajectory alternatives and defending missile beliefs.

Guidance hypothesis chosen	True strategy of the target		
	T_1H_1	T_1H_2	T_1H_3
$M_1T_1H_1$	$Cost_1$	$Cost_2$	$Cost_3$
$M_1T_1H_2$	$Cost_4$	$Cost_5$	$Cost_6$
$M_1T_1H_3$	$Cost_7$	$Cost_8$	$Cost_9$

Note that the number of waypoints, W, is assumed to be equal to $3P$. We will now use the following notation to name engagements (i.e. guidance hypotheses):

$$M_iT_jH_k \tag{1}$$

This means that guidance is derived for Missile i $(1 \geq i \geq N)$ against Target j $(1 \geq j \geq P)$ under assumption that the target follows Hypothesis k $(1 \geq k \geq W)$.

Another useful notation is on the other hand:

$$T_jH_k \tag{2}$$

This is used to name which target actually follows which hypothesis. Therefore, T_jH_k implies target j is following hypothesis k. Note that the hypotheses reflect the three target trajectory classes. Thus, each oncoming threat can perform H_1, H_2 and H_3. Missile guidance can be derived against all target hypotheses of each threat and it is named the guidance hypotheses.

In order to assess the guidance hypotheses, a cost matrix is constructed in this study. Let us consider an engagement involving one missile and one oncoming threat. The cost matrix for the engagement scenario is represented in Table 1. As shown in Table 1, there exist three possible guidance strategies corresponding to the three possible target trajectory hypotheses, $M_1T_1H_1$, $M_1T_1H_2$, and $M_1T_1H_3$. As the target may behave differently from the expected hypothesis, we compute 3 costs corresponding to individual hypotheses H_1, H_2, and H_3, for each guidance strategy. Therefore, for the engagement scenario assumed, 9 costs consist the cost matrix. As the number of missiles and targets increase, the size of the matrix will grow accordingly.

5. Cost computation

Costs are computed through trajectory extrapolations and NEZ considerations. Target trajectories are extrapolated as explained previously using TSG and PN. Missile trajectories are extrapolated using PN guidance on a Predicted Interception Point (PIP); however, other mid-course guidance laws such as DGGL in Shin et al. (2010b) can be considered. Coordinates of the PIP are computed based on the following equation:

$$XYZ_{PIP}(t) = XYZ_T(t+t_{go}) \tag{3}$$

where XYZ_T are the target coordinates, in inertial frame. Note that this paper considers the air defence problem in 3D, so the nature of the problem and the models for the various movements of the vehicles are represented in 3D. For instance, as shown in Eq. (3), position vectors are represented in 3D. Here, the time to go, t_{go}, is obtained using the following equation:

$$t_{go} = \frac{R_{MT}}{V_c} \tag{4}$$

where R_{MT} is the missile-threat distance and V_c is the closing velocity. From the current positions and velocities of the missile and threat, t_{go} can be easily obtained. Then, the coordinates of the target using the trajectory hypothesis selected at time $t+t_{go}$ can be computed, i.e. the right-hand side of Eq. (3) is obtained. Substituting these coordinates into Eq. (3) yields the positions of the PIP. For every time sample of the targets trajectory, the PIP coordinates are calculated. Instead of calculating PN guidance commands against current motion information of the threat, the missile is guided to the PIP using the PN guidance law and missile states at the next sampling time are updated. For deriving guidance commands, the velocity and acceleration of the PIP are assumed to be zero. For initial extrapolations, i.e. when missiles are not already in flight, it is assumed that their velocity vector is aimed directly at the waypoint of the selected hypothesis. This is also used in the model when actually shooting missiles. PN on PIP objective makes use of the assumed knowledge of the targets behaviour and allows the SENEZ target allocation algorithm to launch several defending missiles against the same real threat following different mid-course paths. The SENEZ principle is indeed to shot multiple missiles to anticipate targets behaviour such as doglegs and new target detections.

In this paper, waypoints are simply generated for each trajectory hypothesis of each target. As PIPs are computed for each waypoint, the number of PIPs is equivalent to the number of waypoints, i.e. the number of the trajectory sectors multiplied by the number of oncoming threats. Note that since PIPs are determined from the threat behaviour hypotheses and the corresponding waypoints, how to select waypoints is of great importance. Therefore, this paper investigates how the waypoint position influences the performance of the proposed allocation and guidance scheme via Monte Carlo simulations. The distance and angle of the waypoint in the relative polar coordinate change in the Monte Carlo simulation and whether or not the opposed scheme is still able to intercept the threat is examined. Note that it should be possible to select these waypoints in an optimal manner. However, proposing methods to effectively or optimally select these waypoints are not subject of this paper, but of future study.

Once missile trajectories have been computed, the costs are evaluated. The NEZ concept is applied as well as a modelling of the field of view of the missiles seeker. Two zones are defined; the first zone determines if a target can be locked by the seeker (information); the second zone determines if the target can be intercepted (attainability). The cost is simply the relative time when the target enters the intersection of both zones. If it never happens, the cost value is infinite. If the threat is already in both zones at the first sample time, the cost is zero.

When guiding on a hypothesis such as $M_1T_1H_1$, it is supposed that the seeker always looks at the predicted position of threat T_1 following hypothesis H_1. This gives at every sample time the aiming direction of the seeker. This seeker direction is tested against all other hypotheses to check if a target is within the field of view at this sample time. For example, when the seeker's aiming direction is determined against the threat T_1, following the hypothesis H_1, the test is to check that the trajectory of the threat T_1, for another hypothesis H_i, also enters the seeker's field of view. If positive, an interception test using the NEZ evaluates whether interception is possible. As soon as a target enters the field of view and becomes reachable for a hypothesis, the cost is updated to the trajectories current time. The cost computation concludes when all costs, i.e. of all hypotheses, have been computed, or when the last trajectory sample has been reached. This cost logic has been chosen because of the following:

- It takes into account what the missile can or cannot lock on (seeker cone).
- It takes into account the missiles ability to reach the threats (NEZ).

- In most cases, it can be assumed that low costs imply short interception times.

6. Matrix game target allocation algorithm

After costs have been computed, the algorithm has to find the best possible allocation plan. This means we need to construct allocation plans and combine costs. Consider the following illustrative example. One threat T_1 attacks one objective, with three possible hypotheses H_1, H_2 and H_3. Two missiles M_1 and M_2 are allocated to this target. First, it is necessary to determine the possible combinations, excluding options where the two missiles cover the same target hypothesis. We also compute the cost matrix of each missile as described in Sections 4 and 5. Remember that low cost values imply hopefully early interceptions. Infinite values mean interception is not possible.

Using combinations of min max operators we construct the whole problems cost matrix and advise the best one, mini max game equilibrium (Basar & Olsder, 1982). The best allocation plan, denoted as C_{i^*,j^*}, is the plan that minimises the cost value whatever is the target trajectory.

$$\min_{i,j}(C_{i,j}) = \min_{i,j}\left(\max_k(\min(C_{M_1T_1H_i/T_1H_k}, C_{M_2T_1H_j/T_1H_k}))\right), \tag{5}$$

where i, j are target waypoint beliefs defining the defending missile strategies and k is the waypoint number defining the threat strategies (trajectories). From Eq. (5), it is clear that

$$C_{i,j} = \max_k C_{i,j/k}, \tag{6}$$

where:

$$C_{i,j/k} = \min(C_{M_1T_1H_i/T_1H_k}, C_{M_2T_1H_j/T_1H_k}) \tag{7}$$

An example of this cost matrix of allocation plan is represented in Table 2. For clarity, note that Table 2 is slightly different from Table 1 as, unlike in Table 1, the number of missiles is 2 in Table 2. Values in each cell for T_1H_k, for k=1,2,3, are obtained from $C_{i,j/k}$ represented in Eq. (7). The best allocation plan of this simple case is thus $M_1T_1H_1-M_2T_1H_3$ ($i^*=1$; $j^*=3$) which means guiding M_1 based on hypothesis H_1 of T_1 and M_2 on hypothesis H_3 of the same target. By playing this plan, the second hypothesis is covered with a satisfactory cost of 1.8, and no additional missile is needed.

This algorithm could also be used to optimise the number of missile to be involved: If no satisfactory solution as the costs are higher than a threshold, the procedure can re-start with an additional missile, three missiles in the proposed case.

The same principle applies when there are more than two missiles, and more than one target (the SENEZ algorithm has been written and evaluated in general scenarios). The mathematical formula for the construction and optimisation of allocation plans cost matrix then becomes as follows:

$$\min_{A,B}(C_{A,B}) = \min_{A,B}\left(\max_{i,j}\left(\min_k(C_{M_k,T_{A(k)},H_{B(k)}/T_iH_j})\right)\right) \tag{8}$$

where

Table 2
Allocation plan cost matrix.

Allocation plan chosen	T_1H_1	T_1H_2	T_1H_3	$C_{i,j}$
$M_1T_1H_1-M_2T_1H_2$	1.5	**5.2**	1.0	5.2
$M_1T_1H_1-M_2T_1H_3$	1.5	**1.8**	1.0	1.8
$M_1T_1H_2-M_2T_1H_1$	2.1	**5.5**	1.2	5.5
$M_1T_1H_2-M_2T_1H_3$	**INF.**	1.8	1.0	INF.
$M_1T_1H_3-M_2T_1H_1$	**2.1**	1.8	1.5	2.1
$M_1T_1H_3-M_2T_1H_2$	**INF.**	1.8	1.0	INF.

- k is the missile number (between 1 and N, maximum number of defending missile).
- $A(k)$ is the index of the allocated target (to missile k).
- $B(k)$ is the index of the hypothesis used for target $A(k)$.
- i ($1 \le i \le T$) and j ($1 \le j \le W$) so that T_i is an incoming target and H_j one of the possible hypotheses.

Obviously, when looking for the maximum (in the previous formulae), one scans all possible T_iH_j. An A, B vector pair represents one allocation plan.

To be valid, one allocation plan must comply with the following constraints:

- All incoming targets should appear at least one time in A.
- A target-hypothesis (target number/waypoint number) cannot appear more than one time per allocation plan.

The algorithm has then to find among all possible plans (A, B combinations), the plan that minimises $C_{A,B}$.

By defining heuristics, it is possible to prune potential allocation plans and to focus the algorithm on the most promising solutions (A*, Dijkstra algorithms Shin et al., 2010).

7. Guidance logics

The two diagrams in Figs. 6 and 7 summarise the defending guidance phases (mid-course, homing phase) and explain how the Simulink Common Model operates.

In the mid-course phase, as shown in Fig. 6, target trajectories are extrapolated on every uplink update and extrapolated coordinates are sent to the seeker and the guidance processor. The mid-course phase block also sends PIP coordinates to the guidance subsystem, and conventional target coordinates to the seeker. The guidance law applied during the mid-course phase is the conventional PN. This way the missile is guided on the PIP with PN guidance while looking for the target around the extrapolated position.

When the seeker locks on a target, the missile switches to homing guidance and uses noisy seeker measurements. As represented in Fig. 7, in the homing phase, Kalman filter is applied to estimate the target states from the noisy measurements collected from the seeker. The model used in this filter can be found in the reference above (Zarchan, 2007, Chap. 9). Then, terminal homing guidance computes DGL1 commands based on this state estimate. As shown in Fig. 7, target trajectory extrapolation is not needed in the terminal homing phase.

A seeker subsystem takes into account the dynamics of the antenna loop and outputs noisy measurements of the target position. When the seeker locks on a target, the seeker control loop tries to keep the boresight error as low as possible.

6DOF model, denoted as "Pilot/Airframe/IMU …" in Figs. 6 and 7, involves an autopilot, an environment model (giving pressure, sound velocity and so on), an airframe model, an IMU model as well as models for actuators and propulsion. As can be expected from the full 6DOF model in the guidance block, both the mid-course and homing guidance algorithms are designed in 3D.

8. Simulation study

8.1. Benchmark simulation

Several scenarios for air defence in the ground and naval contexts have been defined in 3D space. A target allocation benchmark policy, with neither re-allocation, nor SENEZ features,

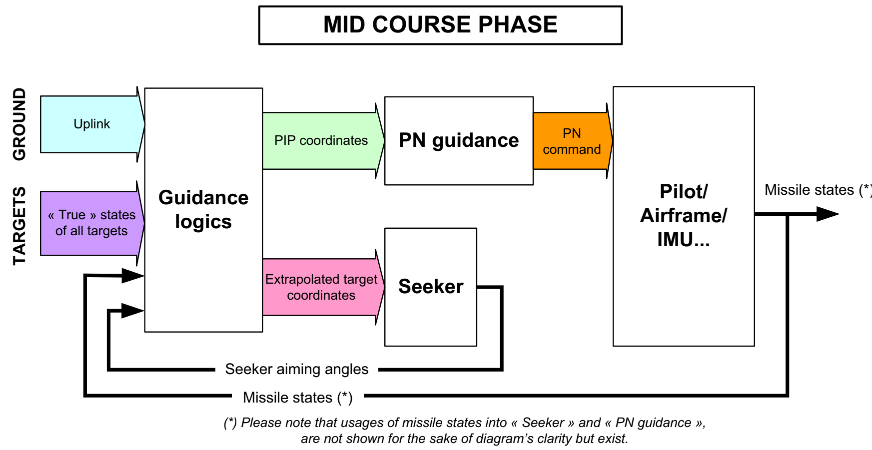

Fig. 6. During mid-course, the guidance logics block extrapolates targets states and PIP coordinates. It also determines if the seeker locks on one of the targets.

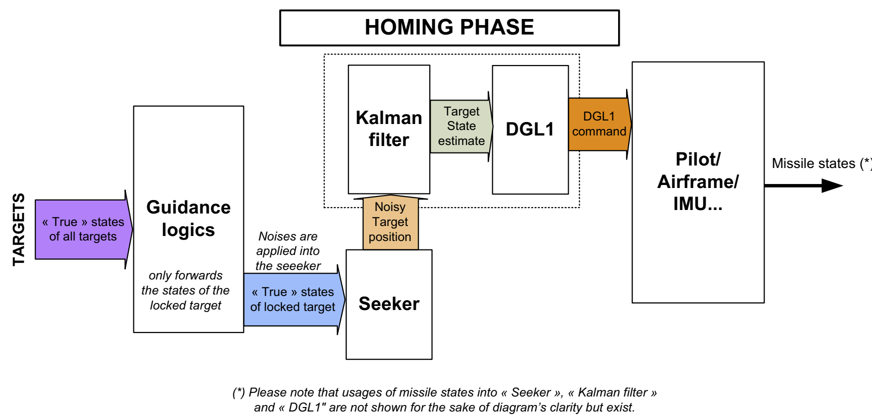

Fig. 7. In homing phase, guidance logics sends true states of locked target. The seeker block applies noises for measurement computation. Kalman filter estimates targets states. Finally a DGL1 in Shima, Shinar, and Weiss (2003) command is applied.

has been defined for comparison purpose. The trajectories of this benchmark simulation on Scenario 3 are illustrated in Fig. 8. As shown in Fig. 8, Scenario 3 deals with ground defence where Air Defence Units (ADUs) are located around (red circle) the objective to be protected (blue diamond). A threat aircraft launches a single missile and then escapes the radar zone. The aircraft and missile are supersonic.

The benchmark policy consists in launching a defending missile as soon as a threat appears in the radar detection range. The benchmark algorithm starts by launching one missile on the merged target. When both targets split, a second missile is shot. This second defending missile will intercept the attacking missile. Due to the sharp escape manoeuvre of the aircraft the first defending missile misses the aircraft. After missing the aircraft, the benchmark algorithm launches a third missile to chase the escaping aircraft. This last missile never reaches its target.

8.2. SENEZ results

When the aircraft crosses the RADAR range the SENEZ algorithm launches two defending missiles (Fig. 9). In ground scenarios, several ADUs are considered, the algorithm automatically deciding by geometric considerations which ADU to use when launching defending missiles. For simplicity, in naval and ground scenarios only one location is considered as the final target goal (ground objective to protect, blue diamond). Simple waypoints are used to generate target trajectory assumptions, even if it is possible to extend the concept to more sophisticated target trajectory assumptions.

Fig. 9 shows the simulation results of the SENEZ algorithm. As shown in Fig. 9, for a better representation, simulation results are depicted in 2D. By comparing the results represented in this figure with those in Fig. 8, it is possible to examine what happens when using the SENEZ algorithm and what the improvements with respect to the benchmark policy are. The defending missiles are in green and in cyan colours. The aircraft trajectory is in the red line turning on the right side. The magenta line is the trajectory of the missile launched by the aircraft. The defending missiles intercept when the threat trajectories switch from plain to dot lines. The dot lines describe what happens when using the benchmark policy in place of the SENEZ algorithm. The dot lines in black are the target trajectory assumptions continuously refined during the engagement. A straight line assumption was considered by the algorithm, however defended missiles assigned to the right and to the left threats are enough to cover the three waypoint assumptions elaborated when the initial threat appears. The SENEZ algorithm intercepts the attacking missile at longer distance than the benchmark algorithm, around a 1 km improvement. Moreover, SENEZ only launches two defending missiles and also intercepts the launching aircraft which the benchmark algorithm fails to do. The fact that SENEZ directs missiles to the left and right sides, plus the fact that SENEZ launches earlier than the benchmark explains the SENEZ performance improvement.

Monte-Carlo runs have been executed for all the scenarios, comparing interception times obtained with the benchmark model to those obtained with the SENEZ. Disturbances for these runs were as follows:

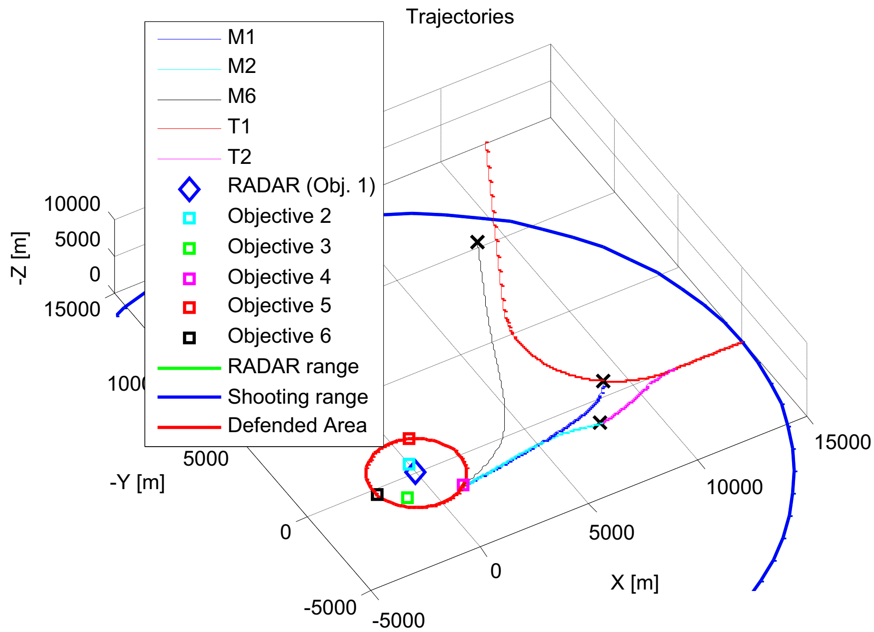

Fig. 8. Trajectories of benchmark simulation on Scenario 3.

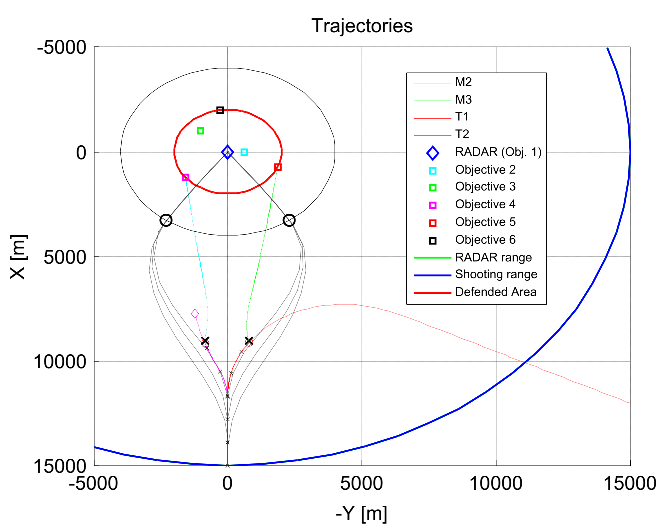

Fig. 9. SENEZ target allocation algorithm on Scenario 3: a top view (2D) of the result in 3D.

- Seeker noise.
- Initial position of the targets (disturbance with standard deviation equal to 50 m).
- Initial Euler angles of the target (disturbance with standard deviation equal to 2.5).

The number of waypoints and their locations are designer's parameters and have not been optimised by an optimisation process. For this reason, parametric sensitivity simulations have been performed on benchmark scenarios. Parametric studies have been conducted on the following aspects:

- Waypoint placements.
- The drag coefficient of the defensive missiles: 0.15, and 0.45 (standard 0.30).
- The seeker range: 3000–5000 m (standard 4000 m).
- The Seeker Field Of View (FOV): 3 and 5 (standard 4).
- Plus some variations on the scenario definition: the time where the second target appears was increased by 1.5 s, then decreased by 1.5 s.

Numerous simulations were run with different R_{wpt} values and different Ψ_{wpt} values (parameters defining the waypoint placement). R_{wpt} radius values between 2 and 20 km have been tested for Ψ_{wpt} varying between 0 and 90. The drawing in the central part of Fig. 10 titled reference describes the successful placements of waypoints located on the left side. The waypoint on the right side is symmetric respect to the horizontal axis. The centre waypoint is then on the horizontal axis (same value for R_{wpt}; and Ψ_{wpt} equal to 0). The horizontal axis is defined by the line of sight between the centre of the area to protect (keep out zone) and the first position of the incoming threat. The waypoints used for managing a salvo of defending missiles are defined when launching a salvo and remain the same as long as the associated target has not been destroyed. Boxes painted with light blue colour correspond to successful interceptions in Monte Carlo. The other drawings explain how the results change respect to the reference configuration. A box in blue colour means that waypoint placements remain valid. When the colour of a box is red it means that this configuration is no more a successful one. Green colour is used for explaining that a placement which was tagged as a bad one in the reference configuration is now becoming an acceptable configuration. In conclusion, we can say that the boxes coloured in blue in the drawings of Fig. 10 are the stable choices for positioning waypoints.

Performance analyses have been executed on various other scenarios for ground and naval applications contexts. Attention is also paid to finding waypoint placements that would be convenient for all ground to air scenarios, or all surface-to-air scenarios. The placement of the waypoints highly depends on the scenario. This tends to prove there would be an advantage in increasing the number of waypoints/missiles corresponding to an increased number of SENEZ hypotheses and/or to optimise the waypoint placement with respect to the threats characteristics.

Potential benefits were first illustrated on all the scenarios considered against targets performing highly demanding evasive manoeuvres as well as apparent single targets that resolve into two splitting targets. The trajectories obtained gave a better idea of the

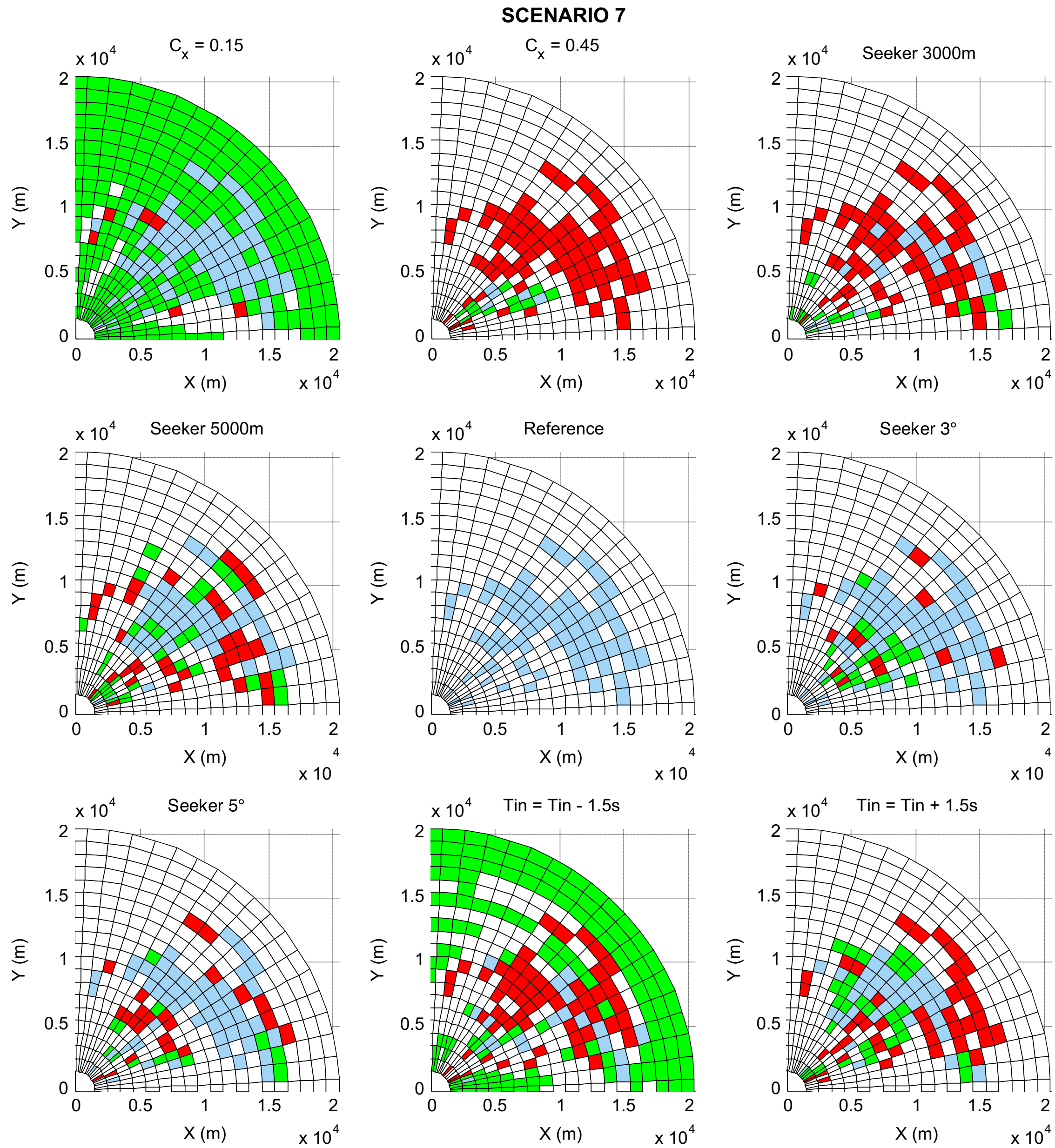

Fig. 10. Sensitivity analysis for Scenario 3 respect to waypoint placements and respect to model parameters; boxes in blue are related to waypoint configurations which remain successful interceptions all along the Monte Carlo tests; boxes in green and in red are successful scenarios for some configurations only. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

SENEZ behaviour. However, the way that target hypotheses are issued proved to be critical. This has been demonstrated by the parametric studies as placement of the waypoints changed greatly the results from one scenario to another. The sensitivity to parameters such as drag and seeker features has also been investigated. Results obtained during these parametric studies seem to show the initial number of waypoints/hypotheses per target chosen three might be too low.

Statistical studies have also been conducted. While providing improved performances in terms of time of last interception in most cases, the standard deviation greatly increased in some scenarios due to misses among the first salvo. These misses may be due to the simplified Kalman estimator used in our model, the chosen mid-course guidance (classical PN on PIP for these tests), the logics used for seeker pointing, or an insufficient number of waypoints.

9. Conclusion

Cooperative guidance is a technique which is likely to emerge as a technology in future weapon systems. Future weapon system scenarios will include the need to engage multiple threats which places greater demands on the guidance chain compared with one-on-one. This project has developed various component technologies supporting the concept of cooperative guidance.

For the terminal phase, differential game guidance laws were applied where the no escape zone was used to characterise the ability of the missile to capture the target. The focus of this paper is concentrated on the way in which some of these technologies are combined to provide an enhanced capability when salvos are launched to deal with target threats, the SENEZ concept. Allocation algorithms have been extended to consider the future possible behaviour of the target; the technique can determine how many

missiles to fire and provide the initialisation for the missiles in the salvo.

Initial results have demonstrated the potential of the SENEZ concept in some cases where this technique has produced results that were better than the baseline allocation algorithm. Although the potential has been demonstrated it remains to examine the full robustness of the approach in terms of range of scenarios and optimisation of parameter setting.

10. SENEZ perspectives

SENEZ guidance attempts to embed the future possible target behaviour into the guidance strategy by using goal oriented predictions of partitioned threat trajectories to drive missile allocation and guidance commands. As such the SENEZ approach offers an alternative to mid-course guidance schemes which guide the intercepting missile or missiles towards a weighted track. The general application of SENEZ would lead to a major change in the current weapon C^2 philosophy including the target assignment and guidance algorithms for naval applications which may not be justifiable.

A set of hypotheses is valid for a specific class of target, but attacking missiles and target aircrafts have different class of targets because of difference in their manoeuvre levels. For this reason, it could be interesting to improve the approach of this paper by updating the waypoint selection process based on an optimisation procedure which takes into account the threat features (velocity, acceleration capability, etc.).

The SENEZ engagement plan requires a missile to be fired at each partitioned set of trajectories. This is different from many existing naval firing policies which would fire a single missile to the target at long range and would delay firing another missile until later when, if there were sufficient time, a kill assessment would be undertaken before firing a second round. Depending on the evolution of target behaviour, current C^2 algorithms may fire a second missile before the potential interception by the first missile. So existing systems tend to follow a more sequential approach, the naval platform needing to preserve missile stocks so that salvo firings are limited; unlike air platform the naval platform cannot withdraw rapidly from an engagement. The proposed engagement plan is purely geometric in formation as opposed to current schemes which use probabilities that the target is heading for a particular goal. This latter type of engagement plan will generally result in fewer missiles being launched. In the SENEZ scheme, a missile salvo will be fired more often because the potential target trajectories are all equally likely. For instance, when the target is at long range, it is likely that its choice of asset to attack is equiprobable, whereas at the inner range boundary, it is most likely that the target is straight-flying towards its intended target.

Despite these potentially negative assessments of the SENEZ concept, for circumstances where a particularly high value asset such as an aircraft carrier may be targeted and a high probability of successful interception is required, there could be merit using the SENEZ approach instead of C^2 strategies. Essentially, in the naval setting SENEZ may be considered as a possible enhancement for the salvo firing determined by the engagement planning function in existing C^2 systems.

For air-to-air systems the scope for considering a SENEZ form of guidance may be greater. It is often policy for aircraft to fire two missiles at an opposing aircraft engaged at medium range. With a two aircraft patrol, the leader and the wing aircraft will each fire a missile at the target, there is an opportunity to shape the guidance so that possible break manoeuvres are covered. With separate platforms firing the missiles it would be necessary for inter-platform communication so that each missile could be allocated to a unique trajectory partition.

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